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Preface

Special issue on the joint spectral radius: Theory, methods and applications

The joint spectral radius was first defined in 1960 in a paper by G.-C. Rota and G. Strang. It describes the maximal growth rate of arbitrary matrix products of matrices from a given set of matrices. As such it is a natural notion in matrix analysis. It turns out that what is easy to understand for a single matrix, in general poses a number of subtle problems for a set of matrices even if this set only contains two elements.

In the beginning the notion did not receive widespread attention, even though it is intimately related to the stability of time-varying systems and at the time there was considerable interest in this subject. The first edition of Cesari's monograph on stability of ordinary differential equations had just appeared in which this problem has a prominent place. Also Yoshizawa's monograph on the theory of Lyapunov functions only appeared 6 years later. Indeed the intimate link of the theory of the joint spectral radius with stability theory and in particular Lyapunov functions was not noted until in the late 80s Barabanov and also Pyatnitskii took up the study of this problem independently. However, at the time of the first definition of the joint spectral radius this connection was unnoticed.

In the early 90s this all changed. On one hand it was observed, that the question of convergence of products of matrices arises in a natural way in the theory of wavelets. On the other hand time-varying systems were receiving greater attention in the field of control and the study of linear inclusions was prominent. Driven by interest in these two fields, important progress has been made during the last decade. Further interesting applications of the joint spectral radius have been discovered. This quantity turned out to be useful in the analysis of the capacity of certain codes, in the stability analysis of numerical integration schemes and in the study of nonhomogeneous Markov chains. From a theoretical point of view the link between the joint spectral radius and extremal norms was explored in detail, regularity properties of the joint spectral radius as a map were discovered and approximation schemes were shown. Probably the most famous results in this area are related to the *finiteness property*, which was conjectured to hold for all finite matrix sets. This conjecture was later disproved by showing that a counterexample must exist.[1] More on this subject can be found in this issue. Finally, many different approaches to the calculation or maybe better approximation of the joint spectral radius were developed. These range from

[1] In relation to this, the editors would like to ask all future authors to refrain from using the phrase "finiteness conjecture". This is now known to be a false mathematical statement – anything but a conjecture.

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ideas using basic inequalities for the joint spectral radius, the numerical construction of extremal norms, the relation of the joint spectral radius to higher Kronecker powers, to discounted optimal control techniques based on the theory of Lyapunov exponents.

In this special issue we were fortunate to obtain submissions which present advances as well in new developments in the computation of the joint spectral radius as in theoretical issues.

We are grateful to all the contributors to this special issue and we hope that this collection will prove to be useful for researchers interested in the field. Clearly, there remain many open questions and new ones will arise as new applications are discovered.

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